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# Determination of the Breaking Load for Concrete Slabs Based on the Deformation Theory of Plasticity

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**Abstract**

The paper proposes a technique of determining the maximum critical load for concrete slabs based on the deformation theory of plasticity by G.A. Geniev. To solve the problem the method of successive loading is used. Physically nonlinear problem is reduced to successive solution of elastic problems for the construction with a variable elasticity modulus and Poisson's ratio within height. In deriving equations, we take in account cracking in the tension zone of concrete, creep of concrete and nonlinear behavior of steel reinforcement. The work of steel is described by the Prandtl diagram. For the tensile zone, we use Mohr's failure criterion. We perform the solution numerically using the finite difference method. The problem reduces to the fourth-order differential equation for the deflection. We compare our results with the experimental data for square plate, hinged on the contour and uniformly loaded.



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*Keywords:* Concrete slabs; cracks; non-linearity; deformation theory of plasticity; creep.

## 1. Introduction

Concrete is a material with a complex mechanical behavior [1,2,3]. Firstly, it is characterized by a non-linear relationship between stresses and instant deformations. In addition, under long-term effects of load concrete exhibits nonlinear creep [4]. Also, concrete works differently in tension and compression. Accounting of the above factors is a complex problem, so far unsolved not fully. For statically indeterminate bar systems, plates and shells there is a widely used method of limiting equilibrium [4]. Despite its simplicity, this method has a major drawback associated with the fact that you must be aware of the scheme of structural failure. This information can only be obtained by experience.

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For a number of rod systems, mostly massive, stress-strain state is determined using the linear theory of homogeneous isotropic elastic body. There are studies in which the physically nonlinear problem reduces to solving a number of problems for an inhomogeneous elastic body [5-15]. However, they are mainly devoted to the rod systems, as well as thick-walled cylinders.

## 2. Formulation of the problem

We consider the concrete slab, hinged along the contour and loaded by uniformly distributed load $q$ (Fig.1).

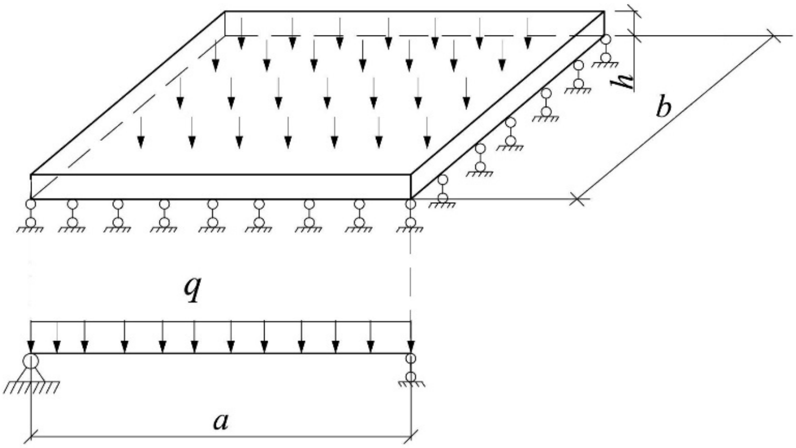


Fig. 1. The settlement scheme

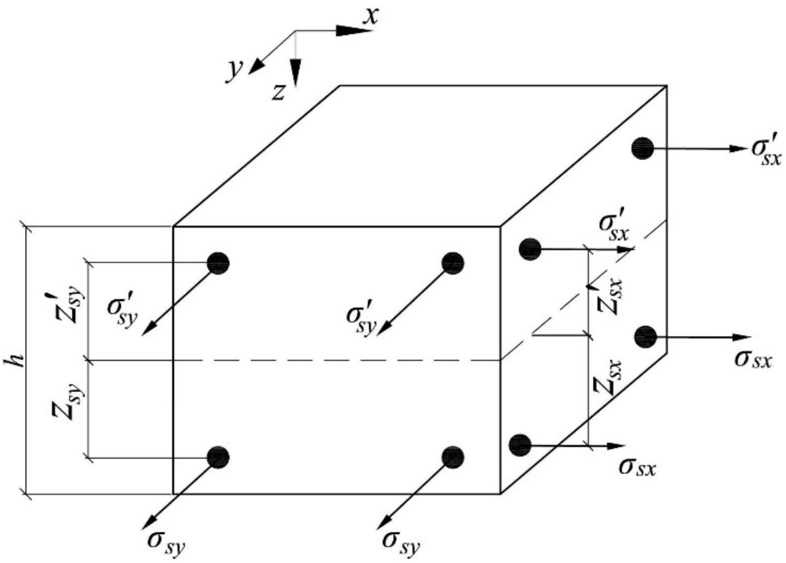


Fig.2. Reinforced concrete slab element

We cut out of a plate element with dimensions dx and dy and consider its equilibrium (Fig.2). In deriving equations we assume that the concrete modulus of elasticity Eb, and the Poisson's ratio ν are functions of x, y, and z. For a concrete link between strains $\varepsilon_{bx}, \varepsilon_{by}, \gamma_{bxy}$ and stresses $\sigma_{bx}, \sigma_{by}, \tau_{bxy}$ can be written as:

$$\begin{aligned}
\varepsilon_{bx} &= \frac{1}{E_b}(\sigma_{bx} - \nu\sigma_{by}) + \varepsilon^*_{bx};\\
\varepsilon_{by} &= \frac{1}{E_b}(\sigma_{by} - \nu\sigma_{bx}) + \varepsilon^*_{by};\\
\gamma_{bxy} &= \frac{\tau_{bxy}}{G_b} + \gamma^*_{bxy},
\end{aligned} \tag{1}$$

where $G_b = \dfrac{E_b}{2(1+\nu)}$ – concrete shear modulus, $\varepsilon^*_{bi}$ – forced deformations, which can include creep deformations, shrinkage and others.

In deriving equations we use the hypothesis of straight normals, according to which the relationship between the deformations and deflection of plate *w* has the form [16,17]:

$$\begin{aligned}
\varepsilon_{bx} &= \varepsilon_{bx0} - z\frac{d^2 w}{dx^2};\\
\varepsilon_{by} &= \varepsilon_{by0} - z\frac{d^2 w}{dy^2};\\
\gamma_{bxy} &= \gamma_{bxy0} - 2z\frac{d^2 w}{dxdy},
\end{aligned} \tag{2}$$

where $\varepsilon_{bx0}$, $\varepsilon_{by0}, \gamma_{bxy0}$ – strains in the middle surface.

We express the stresses across the strains from (1):

$$\begin{aligned}
\sigma_{bx} &= \frac{E_b}{1-\nu^2}(\varepsilon_{bx} + \nu\varepsilon_{by} - (\varepsilon^*_{bx} + \nu\varepsilon^*_{by})) = \frac{E_b}{1-\nu^2}\left[(\varepsilon_{bx0} + \nu\varepsilon_{by0}) - \mathrm{z}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) - (\varepsilon^*_{bx} + \nu\varepsilon^*_{by})\right];\\
\sigma_{by} &= \frac{E_b}{1-\nu^2}(\varepsilon_{by} + \nu\varepsilon_{bx} - (\varepsilon^*_{by} + \nu\varepsilon^*_{bx})) = \frac{E_b}{1-\nu^2}\left[(\varepsilon_{by0} + \nu\varepsilon_{bx0}) - \mathrm{z}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) - (\varepsilon^*_{by} + \nu\varepsilon^*_{bx})\right];\\
\tau_{bxy} &= \frac{E_b}{2(1+\nu)}(\gamma_{bxy} - \gamma^*_{bxy}) = \frac{E_b}{2(1+\nu)}(\gamma_{bxy0} - 2z\frac{\partial^2 w}{\partial x \partial y} - \gamma^*_{bxy}).
\end{aligned} \tag{3}$$

Bending moments $M_x$ and $M_y$ are compounded by moments $M_{bx}$ and $M_{by}$, perceived by concrete, as well as moments $M_{sx}$ and $M_{sy}$, perceived by reinforcement:

$$\begin{aligned}
M_x &= M_{bx} + M_{sx};\\
M_y &= M_{by} + M_{sy}.
\end{aligned} \tag{4}$$

Moments $M_{bx}$ and $M_{by}$ are defined as follows:

$$M_{bx} = \int_{-h/2}^{h/2} \sigma_{bx} z dz = \varepsilon_{bx0} C_{b1} + \varepsilon_{by0} C_{b2} - \frac{\partial^2 w}{\partial x^2} D_{b1} - \frac{\partial^2 w}{\partial y^2} D_{b2} - M_{bx}^*;$$
$$M_{by} = \int_{-h/2}^{h/2} \sigma_{by} z dz = \varepsilon_{by0} C_{b1} + \varepsilon_{bx0} C_{b2} - \frac{\partial^2 w}{\partial y^2} D_{b1} - \frac{\partial^2 w}{\partial x^2} D_{b2} - M_{by}^*, \quad (5)$$

where $C_{b1} = \int_{-h/2}^{h/2} \frac{E_b}{1-\nu^2} z dz,\ C_{b2} = \int_{-h/2}^{h/2} \frac{E_b \nu}{1-\nu^2} z dz,\ D_{b1} = \int_{-h/2}^{h/2} \frac{E_b}{1-\nu^2} z^2 dz,\quad D_{b2} = \int_{-h/2}^{h/2} \frac{E_b \nu}{1-\nu^2} z^2 dz,$

$$M_{bx}^* = \int_{-h/2}^{h/2} \frac{E_b}{1-\nu^2} \left[ \varepsilon_{bx}^* + \nu \varepsilon_{by}^* \right] z dz,\quad M_{by}^* = \int_{-h/2}^{h/2} \frac{E_b}{1-\nu^2} \left[ \varepsilon_{by}^* + \nu \varepsilon_{bx}^* \right] z dz.$$

Moments perceived by reinforcement are calculated as follows:

$$M_{sx} = h(\sigma_{sx} z_{sx} \mu_{sx} - \sigma'_{sx} z'_{sx} \mu'_{sx});\quad M_{sy} = h(\sigma_{sy} z_{sy} \mu_{sy} - \sigma'_{sy} z'_{sy} \mu'_{sy}), \quad (6)$$

where $\mu_{sx}$, $\mu'_{sx}$, $\mu_{sy}$, $\mu'_{sy}$ – reinforcement ratios which are equal to the quotient of reinforcement area on 1 meter of slab length to its thickness.

Stress in the reinforcement are determined by the terms of its collaboration with the concrete:

$$\sigma_{sx} = E_s \left( \varepsilon_{bx0} - z_{sx} \frac{\partial^2 w}{\partial x^2} \right); \sigma'_{sx} = E_s \left( \varepsilon_{bx0} + z'_{sx} \frac{\partial^2 w}{\partial x^2} \right); \sigma_{sy} = E_s \left( \varepsilon_{by0} - z_{sy} \frac{\partial^2 w}{\partial y^2} \right); \sigma'_{sy} = E_s \left( \varepsilon_{by0} + z'_{sy} \frac{\partial^2 w}{\partial y^2} \right). \quad (7)$$

The deformations of the middle surface $\varepsilon_{bx0}$, $\varepsilon_{by0}$, $\gamma_{bxy0}$ can be found from the condition that the longitudinal forces in the element are equal to zero:

$$N_x = \int_{-h/2}^{h/2} \sigma_{bx} dz + h(\sigma_{sx}\mu_{sx} + \sigma'_{sx}\mu'_{sx}) = \varepsilon_{bx0}(B_{b1} + B_{sx}) + \varepsilon_{by0} B_{b2} - \frac{\partial^2 w}{\partial x^2}(C_{b1} + C_{sx}) - \frac{\partial^2 w}{\partial y^2} C_{b2} - N_{bx}^* = 0;$$
$$N_y = \int_{-h/2}^{h/2} \sigma_{by} dz + h(\sigma_{sy}\mu_{sy} + \sigma'_{sy}\mu'_{sy}) = \varepsilon_{by0}(B_{b1} + B_{sy}) + \varepsilon_{bx0} B_{b2} - \frac{\partial^2 w}{\partial y^2}(C_{b1} + C_{sy}) - \frac{\partial^2 w}{\partial x^2} C_{b2} - N_{by}^* = 0; \quad (8)$$
$$N_{xy} = \int_{-h/2}^{h/2} \tau_{bxy} dz = \gamma_{bxy0} B_{b12} - 2C_{b12} \frac{\partial^2 w}{\partial x \partial y} - N_{bxy}^* = 0,$$

where $C_{b12} = \int_{-h/2}^{h/2} G_b z dz,\ B_{b1} = \int_{-h/2}^{h/2} \frac{E_b}{1-\nu^2} dz,\ B_{b2} = \int_{-h/2}^{h/2} \frac{E_b \nu}{1-\nu^2} dz,\quad B_{sx} = hE_s(\mu_{sx} + \mu'_{sx}),\quad B_{sy} = hE_s(\mu_{sy} + \mu'_{sy}),$

$$B_{b12} = \int_{-h/2}^{h/2} G_b z dz,\ N_{bx}^* = \int_{-h/2}^{h/2} \frac{E_b}{1-\nu^2}\left[ \varepsilon_{bx}^* + \nu \varepsilon_{by}^* \right] dz,\quad N_{bxy}^* = \int_{-h/2}^{h/2} \frac{E_b}{1-\nu^2}\left[ \varepsilon_{by}^* + \nu \varepsilon_{bx}^* \right] dz,\quad N_{xy}^* = \int_{-h/2}^{h/2} G_b \gamma_{bxy}^* dz.$$

Torque is defined as follows:

$$H=\int_{-h/2}^{h/2}\tau_{bxy}zdz=\gamma_{bxy0}C_{b12}-2\frac{\partial^2 w}{\partial x\partial y}D_{b12}-H_b^*=0,\ where\ D_{b12}=\int_{-h/2}^{h/2}G_b z^2dz,\ H_b^*=\int_{-h/2}^{h/2}G_b\gamma_{bxy}^*zdz.$$

Expressing from (8) deformations of the middle surface and substituting them into the expression for the bending and twisting moments, we get:

$$M_x=-a_{11}(x,y)\frac{\partial^2 w}{\partial x^2}-a_{12}(x,y)\frac{\partial^2 w}{\partial y^2}-M_x^*;\ M_y=-a_{21}(x,y)\frac{\partial^2 w}{\partial x^2}-a_{22}(x,y)\frac{\partial^2 w}{\partial y^2}-M_y^*;$$
$$H=-a_{33}(x,y)\frac{\partial^2 w}{\partial x\partial y}-H^*. \quad (9)$$

The expressions for stiffness coefficients $a_{ij}$ and moments $M_x^*$, $M_y^*$, $H^*$, are not mentioned here because of their bulkiness. The equilibrium equation is of the form [16]:

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 H}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}=-q(x,y). \quad (10)$$

Substitution of (9) into (10) yields to the following differential equation:

$$\frac{\partial^2}{\partial x^2}\left(a_{11}\frac{\partial^2 w}{\partial x^2}+a_{12}\frac{\partial^2 w}{\partial y^2}\right)+2\frac{\partial^2}{\partial x\partial y}\left(a_{33}\frac{\partial^2 w}{\partial x\partial y}\right)+\frac{\partial^2}{\partial y^2}\left(a_{21}\frac{\partial^2 w}{\partial x^2}+a_{22}\frac{\partial^2 w}{\partial y^2}\right)=q-\left(\frac{\partial^2 M_x^*}{\partial x^2}+\frac{\partial^2 M_y^*}{\partial y^2}+2\frac{\partial^2 H}{\partial x\partial y}\right). \quad (11)$$

## 3. Solution of the problem

Equation (11) is solved using the finite difference method. We introduce the grid by *x, y* and *z*. The integrals in the expression for the coefficients are calculated numerically using the trapezoidal method. We use for concrete deformation theory plasticity by G.A. Geniev [4], according to which the relationship between the strains and stresses of concrete for the plane stress is:

$$\varepsilon_x=\frac{1}{E(\Gamma)}(\sigma_x-\nu\sigma_y)-\frac{g_0\Gamma^2}{3};\quad \varepsilon_y=\frac{1}{E(\Gamma)}(\sigma_y-\nu\sigma_x)-\frac{g_0\Gamma^2}{3};\quad \gamma_{xy}=\frac{\tau_{xy}}{G(\Gamma)}, \quad (12)$$

where $\Gamma=\sqrt{\frac{2}{3}}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2}$ – shear strain rate, $g_0$ – dilation module.

Dilated deformation associated with an increase of volume because of cracks formation, can be regarded as forced. Increase of pressure is performed incrementally, by small portions. At each step, after determining the displacements we calculated stresses and main deformations. Then we determined shear modulus by the deformations. The tangent shear modulus in theory of G.A. Geniev is defined as follows:

$$G_t=G_0(1-\Gamma/\Gamma_s),$$

where $\Gamma_s$ – ultimate intensity of shear strain, $G_0$ – the initial value of the shear modulus.

For the tensile zone we used Mohr failure criterion. When the equivalent stress at some point exceeds the concrete tensile strength $R_{bt}$, the tangent modulus of elasticity in the next step is reset.

## 4. Results and discussion

The problem was solved for a square hinged along the contour plate with the following initial data: size 2x2 m, $h = 12.2$ cm, $\nu = 1/6$, $E_{b0} = 2 \cdot 10^4$ MPa, $E_S = 2 \cdot 10^5$ MPa, $\Gamma_s = 0.583 \cdot 10^{-3}$, $R_{bt} = 1$ MPa, $\mu_{sx} = \mu_{sy} = 0.333\%, \mu'_{sx} = \mu'_{sy} = 0$. The test results of the plate are given in the works of N.I. Karpenko [2,3]. In Figure 3, a solid line shows obtained by the authors dependence of the deflection of plate in the center on the load $q$. The dashed lines correspond to the experimental data. The experimental results are very close to the theoretical data.

## 5. Conclusions

Obtained equations and developed technique allows calculation of reinforced concrete slabs, not only in the short term, but also on the long-term effects taking into account creep. Thus, creep law can be set in an arbitrary manner. The coincidence of the theoretical results with the experimental data indicates the reliability of methods.

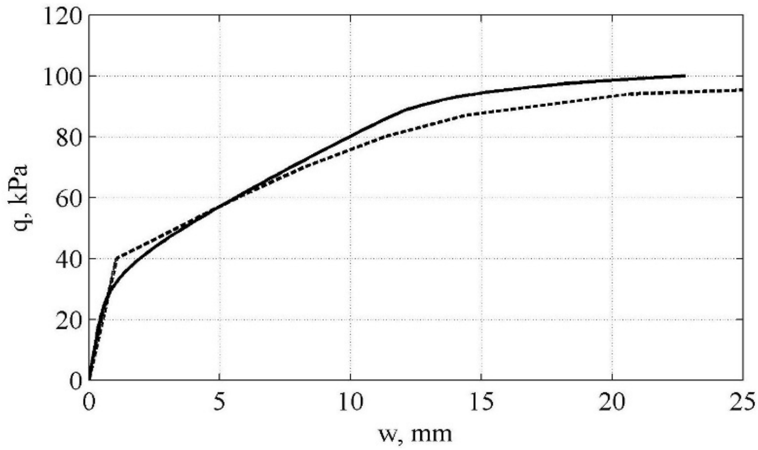


Fig. 3. The dependence of the load on deflection: dashed line - experiment, solid line - calculation

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